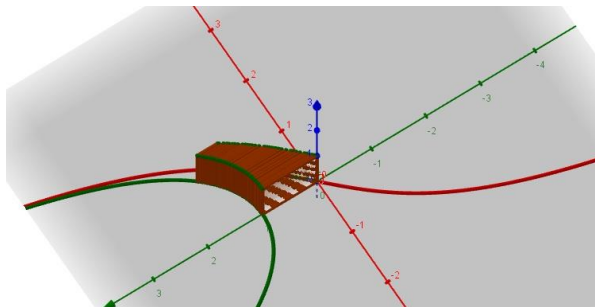


Math 005B Exam #2 – Illustrated version. On Canvas, you can find some GeoGebra applets that go with the pictures.

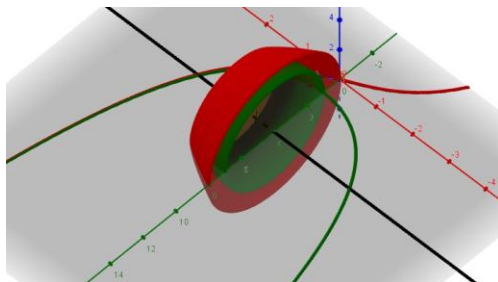
Instructions – this is open book and notes, but **you must show all of your work**. This is due by Friday, May 22.

1. (10 pts each) Let $f(x) = \cosh(x)$ and $g(x) = \sinh(x)$, $a = 0$ and $b = 1$.

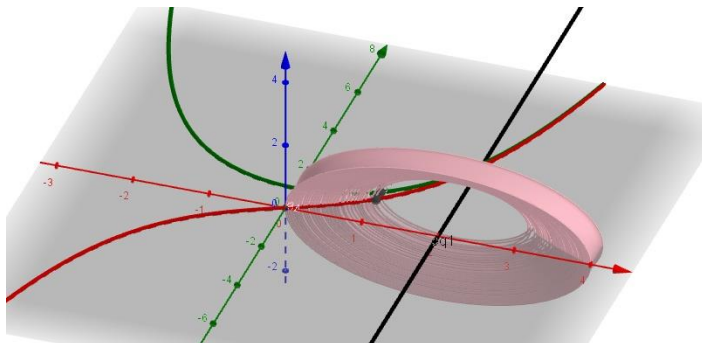
A) Find the volume of the solid with base on the xy plane, bounded by the region above, whose cross-sections perpendicular to the x axis are squares.



B) Find the volume of the solid formed if the region above is rotated about the line $y = 4$.



C) Find the volume of the solid formed if the region above is rotated about the line $x = 2$.



Hint: It might be easiest to convert all of this to exponential functions – or, feel free to use hyperbolic trig identities, if you show where you use them (as always, show your steps!)

2. (15 pts)

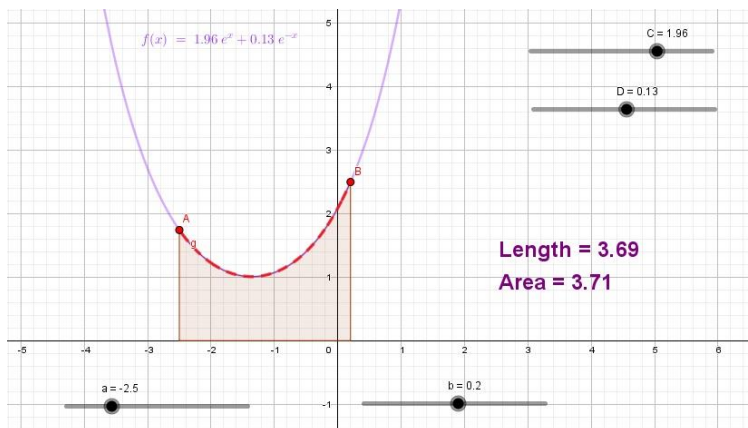
A) Find $\int \frac{x^4}{x^3 - x^2 + x - 1} dx$ – Show all your steps.

B) Find $\int_0^2 \frac{x^4}{x^3 - x^2 + x - 1} dx$ – Show all your steps.

3. (15 pts)

A) Let $f(x) = \cosh(x)$. Show that, on any interval $[a, b]$, the arclength of $f(x)$ in units, is equal to the area under $f(x)$ in square units.

B) Let $g(x) = Ce^x + De^{-x}$. Find a formula for D in terms of C , such that on any interval $[a, b]$, the arclength of $g(x)$ in units, is equal to the area under $g(x)$ in square units. (Obviously, from part “A”, $C = D = \frac{1}{2}$ is one such possibility. You need to find a formula for all of them.)

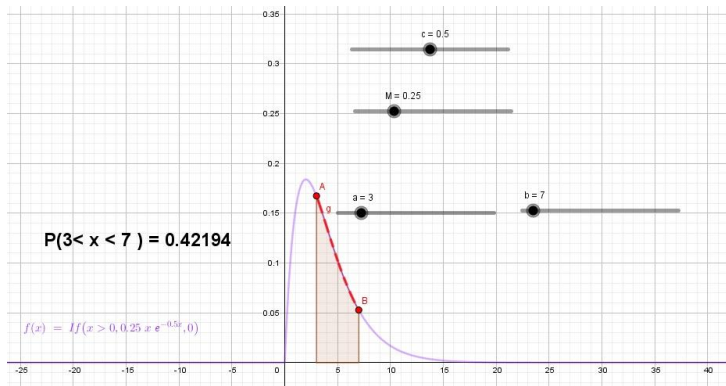


4. 30 pts:

A) Warm-up: Calculate $\int x e^{-cx} dx$ and $\int x^2 e^{-cx} dx$. You may use the tabular method, if you prefer. Show your work! Check your answers to make sure they are correct, using whatever software or internet resources you prefer. **You will need these answers for what follows, and in what follows YOU DO NOT NEED TO RECALCULATE THESE INTEGRALS – YOU MAY USE THEM and evaluate them or take limits, as appropriate.**

Now, for the rest of the problem: Let $f(x) = \begin{cases} 0, & x < 0 \\ M x e^{-cx}, & x \geq 0 \end{cases}$.

Wherever you need to calculate an improper integral in what follows, you must set it up properly, in terms of limits, and show your steps in evaluating those limits.



B) Find the value of M , in terms of “ c ,” such that $f(x)$ is a probability distribution (see example #2, p. 615, section 8.5, if you need help.)

C) Find the mean (expected value) of $f(x)$, in terms of “ c .”

Now, suppose that when a person is infected with a virus, the time before displaying symptoms is distributed as $f(x)$, with a mean time of 6 days.

D) Write out $f(x)$ explicitly (giving actual values for “ M ” and “ c .”)

E) Find $P(x < 6)$.

F) Find $P(5 < x < 7)$.

G) Find $P(x > 10)$.

H) Find the median of the distribution – i.e. the time, T , such that $P(x < T) = 0.5$ to 3 decimal places. Use calculus and show your steps!

I) The mode is defined to be the location of the maximum value of $f(x)$, if it exists. Find the mode of this distribution to 3 decimal places. Again, use calculus and show your steps!

5. (10 pts) A steel plate is suspended in water. The shape of the plate is the base of a parabola, with a focal length of P meters, cut off at its latus rectum. The vertex of the parabola is at the surface of the water, and the parabola opens downward. Find the hydrostatic force on the plate, in Newtons (see example #1, p. 599 to get the units correct.), in terms of P . You'll need to look up what the latus rectum of a parabola is. Also, it's probably best to put the origin at the surface of the water, and positive y facing downward.

(The picture gives away the meaning of "latus rectum....")

